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Introduction to Gauge/Gravity Duality

Lecture 1

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Abstract

A series of four lectures given at the Australian National University in December 2011 for the 2nd Asia-Pacific Summer School in Mathematical Physics/22nd Canberra International Physics Summer School.

1 Outline of the Lectures

The goal of these lectures is to impart some of the ideas of the gauge/gravity duality and to help facilitate reading the fundmental papers [1, 2, 3]. The rough outline of the four lectures is:

Lecture 1: Introduction and some motivation from supergravity/string theory

Lecture 2: The Bulk: geometry of AdS space

Lecture 3: The Boundary: conformal symmetry and quantum field theory

Lecture 4: The gauge/gravity duality and computing boundary correlators from the bulk

The study of gauge/gravity duality might be a natural progression after taking courses in quantum field theory and general relativity. These lectures are intended for advanced undergraduates and beginning graduate students who may or may not have completed courses in quantum field theory/general relativity.

2 Introduction

The two most fundamental developments of 20th century theoretical physics were quantum field theory (QFT) and general relativity (GR). General relativity was discovered by Einstein [4] to resolve the blatent discrepency between Newtonian gravity and special relativity. In this paper alone, the foundation of general relativity was laid out and to this day has vanquished all challengers for the description of relativistic classical gravity. Quantum field theory on the other hand was developed over a far longer period, starting in the late 1930's to early 1940's, and numerous authors contributed even just to the early stages of development. It was not until the late 1960's that quantum field theory as we know it know was broadly accepted as the fundamental description of particle physics but today QFT is applied to an array of problems far beyond particle physics. Despite the many successes of QFT, many foundational issues are still poorly understood and basic questions about non-perturbative definitions of QFT are still active areas of research today.

It is a natural desire to develop a single framework within which one can derive all aspects of fundamental physics. One strategy to unify QFT and GR is through some sort of Grand Unification Theory (GUT). In such a scenario, at high enough energies ($\sim 10^{16}$GeV) the electro-weak and strong nuclear forces will become unified and at energies above this there will be just one gauge coupling. Then towards the Planck scale ($\sim 10^{18}$GeV) this gauge coupling will unify with the gravitational coupling (sometimes called Newtons constant) and there will be just one force of nature. There are many excellent reasons to be optimistic that this is a good model of the high energy behaviour of our universe. The line of thinking behind gauge/gravity however is quite different and is not obviously related to GUT theories.

The basic idea of gauge/gravity duality is that particular quantum field theories in d-dimensions have alternate descriptions as particular theories of quantum gravity in $(d+1)$-dimensions. There are infinitely many examples of quantum field theory, the standard model of particle physics is just the particular model which happens to describe the real world at least up to the energy scale ~ 100GeV. By the same token, there are infinitely many theories of gravity: the graviton can couple to additional matter fields in many ways and work within string theory suggests that in dimensions less than ten, there are infinitely many such theories which are consistent quantum mechanically. The claim of gauge gravity duality is certainly not that *every* quantum field theory can be described by quantum gravity but to some extent, in the other direction we do expect that every consistent theory of quantum theory of gravity on AdS space (to be defined below) to be a QFT in disguise.

Perhaps the most bizarre aspect of the gauge/gravity duality is that the QFT and the gravity theory live in different spacetimes, even different dimensions! As we will see in some detail, these different spacetimes are however not unrelated. At least in the current understanding of gauge/gravity duality, the QFT lives on the boundary of the spacetime where the gravity theory lives. For this reason one frequently hears the term *holography* in reference to a optical hologram, where a 3d image is stored in data on a 2d surface.

Once one has come to peace with the idea of studying physics in a dimension different than where we would live, there is perhaps a more profound aspect of gauge/gravity duality worth expounding. As we all know, a good physicist makes good approximations. Sometimes we can solve physical problems analytically but it is quite rare and unusual for these problems to be truly physically interesting…not to say this never happens, it does, but it is not common. As a result we typically resort to some sort of perturbation theory and for example the Feynmann diagram expansion of the path integral is a particularly ingenious organization of perturbation theory.

Quite often our best formulation of a physical problem fails to admit a perturbative description, usually because the obvious coupling in the theory is not small but $\mathcal{O}(1)$ or larger. This is not so much a failure on our part to *construct* a model which represents the physics but a failure to *solve* the model. For example, it is generally accepted that QCD is a good description of the strong interactions and can be used to compute at high energies but deriving the low energy property of quark confinement from the Lagrangian is difficult due to strong coupling. It is generally accepted that QCD does indeed model quark confinement but it is just difficult to demonstrate this explicitly.

This is where duality can be quite useful: when either theory is weakly coupled, the dual theory is strongly coupled. Unfortunately it does not work the other way around, just because one theory is strongly coupled one cannot assume that the dual theory will be weakly coupled and indeed, this will not be true in general. So in the case of gauge/gravity duality, if you are interested in quantum gravity then for certain problems the dual quantum field theory will allow you to compute outside your usual allowed regime. Likewise if you are a quantum field theorist, you can use quantum gravity as a tool to access regions of your theory whgich previously were perturbatively forbidden. An ambituous way to test this duality would be to find a simple example, solve both theories non-perturbatively and demonstrate their equivalence. It is not unreasonable to expect this to be achievable in very simple examples and for $d = 2$ this is an active area of research.

Some interesting ideas which gauge/gravity duality synthesizes are the following[1] (this is by no means a comprehensive list and is in no particular order):

- provides an explicit realization of an old idea of 't Hooft [5] that in an expansion in N_c, gauge theory could be described by string theory

- provides a concrete realization of duality between open and closed string theory in particular it isolates crucial features of D-branes [6] and the neccesity for their inclusion in string theory

- D-branes provided the basis for counting black hole microstates [7] for a certain class of black holes. The essence of this structure is rooted in gauge/gravity duality.

- black holes in the bulk gravity theory are dual to thermal states in the dual unitary field theory. In this sense gauge/gravity duality provides a strategy to resolve the information paradox of black holes.

- the bulk theory has natural parameters dual to finite density quantum field theory. Inital computations [8] in such states provided key insights into the quark gluon plasma which has been created at the Relatavistic Heavy Ion Collider (RHIC) in Brookhaven Labs, New York. See [9, 10] for reviews.

- The membrane paradigm [11] was suggestive of a connection between black hole physics and fluid mechanics. That has now been made very precise with the relativistic [12] and non-relativistic [13, 14] Navier-Stokes equations being derived from gravity.

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[1] these comments might be beyond the level of the lectures but hopefully will inspire some late night conversations, indeed they were written quite late at night

- the gravity dual can provide a perturbative description to certain confining four dimensional gauge theories [15]

There exist several excellent introductions to AdS/CFT and its applications. A good source of information is the original review [16], other quite general reviews are [17, 18, 19, 20, 21, 22, 23]. More recent articles with a view towards applications to condensed matter and nuclear physics are [24, 25, 26, 27, 10, 28].

3 Type II Supergravity

3.1 Supergravity and a little bit of string theory

The low energy limit of both type IIA and type IIB string theory contains a spin two particle, an anti-symmetric 2-tensor and a scalar field

$$\text{NS} : (G_{\mu\nu}, B_{\mu\nu}, \phi). \tag{1}$$

Due to the properties which these particular fields have on the string worldsheet, this is refered to as the Neveu-Schwarz (NS) sector. In addition, there is another set of fields known as the Ramond-Ramond (RR) sector. In type IIA string theory the RR sector is

$$\text{RR (IIA)} : (F_{(0)}, F_{(2)}, F_{(4)}) \tag{2}$$

along with their magnetic duals, while in type IIB string theory this is

$$\text{RR (IIB)} : (F_{(1)}, F_{(3)}, F_{(5)}). \tag{3}$$

Each such RR field is an anti-symmetric tensor with the index on each field indicating the number of space-time indices. In general there exists conventions in which these RR fields are closed under exterior derivative and are thus field strengths. In which case, locally we have

$$F_{(n)} = dC_{(n)} \tag{4}$$

for some potentials[2] $C_{(p-1)}$.

In flat ten dimensional space these fields are massless. In addition, string theory has infinite towers of arbitrarily massive fields (known as Regge trajectories). These massive fields affect the UV behavior of string amplitudes and play a key role in various calculations which suggest that string theory in flat space is a finite theory. However, for the purposes of AdS/CFT and thus these lectures, we are primarily concerned with the *massless* spectrum of string theory outlined in the preceding paragraph. This is a particularly welcome turn of events as it ultimately means we can address a number of very interesting physical problems in the far simpler setting of various forms of gravity/supergravity: string theory on gneral curved spacetimes is a very challenging subject about which very little is currently known. It could quite likely turn out that the detailed analysis of these problems to the point of comparing precisely with real-world experiments, will require understanding the dynamics of these massive string states as well. This is a daunting task and an active area of research.

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[2] except for $F_{(0)}$ in IIA, known as the Romans mass, which deserves special treatment but will not play any role in these lectures

3.2 The Action and Equations of Motion

Having stated without proof the massless spectrum of type II string theory, we now state without proof that the action for these fields is given by type II supergravity [29, 30]. From here on, we will restrict ourselves to IIB string theory and thus type IIB supergravity. Looking ahead, this is so that we can study four-dimensional QFT as opposed to QFT in various odd dimensions.

The details of IIB supergravity can be somewhat unwieldy so we will just keep the pieces which we need. Consider the action for gravity coupled to a scalar field and a $p+2$-form

$$\mathcal{S}_p = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big[e^{-2\phi}\big(R + 4\partial_\mu\phi\partial^\mu\phi\big) - \frac{1}{2}|F_{(p+2)}|^2\Big] \tag{5}$$

with the Bianchi identity (4). When $-1 \leq p \leq 3$ is odd this is a truncation of IIB supergravity. However for $p = 3$, there is a pesky subtlety that the field strength must be self dual

$$F_{(5)} = *F_{(5)}\,. \tag{6}$$

This algebraic condition cannot be derived from any action and naively it implies the vanishing of the kinetic term[3] in (5). This issue consistently recurs and can be somewhat thorny but an adequate workaround is to simply apply (6) after the equations of motion which one derives from (5).

Despite (5) being a significant truncation of IIB supergravity, it admits the following very interesting solutions [31]

$$\begin{aligned} ds^2 &= h^{-1/2}(r)\big[-f(r)dt^2 + ds_p^2\big] + h^{1/2}\big[f^{-1}dr^2 + r^2 d\Omega_{8-p}^2\big]\,, && (7)\\ e^{\phi} &= h^{(3-p)/4}(r)\,, && (8)\\ h(r) &= 1 + \frac{L^{7-p}}{r^{7-p}}\,, \quad f(r) = 1 - \frac{r_0^{7-p}}{r^{7-p}}\,. && (9)\end{aligned}$$

These solutions are similar in spirit to the Reissner-Nordström black hole solution (see [32] chapter 6) in that they are static (non-rotating), they have a horizon at $r = r_0$ and they carry a charge with respect to an anti-symmetric tensor field

$$\int_{\Omega_{8-p}} *F_{(p+2)} = Q\,. \tag{10}$$

As will become clear, the value of p which we are in fact most interested in is $p = 3$. For the moment we should simply note that for this and only this value, the dilaton ϕ is well behaved at $r = 0$, in fact it is constant. Those who know a little more about string theory will understand that this means that string loop corrections can be made vanishingly small and the background is well defined in string perturbation theory. We will now restrict just to this case of $p = 3$.

In the solutions (8) there are two dimensionless parameters which we can tune:

$$s_1 = \frac{L^4}{\kappa_{10}},\quad s_2 = \frac{r_0^4}{\kappa_{10}}\,. \tag{11}$$

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[3] this issue is common to theories of a F_{n+2} form in dimension $d = 4n-2$

Roughly speaking, s_1 controls the temperature of the black brane while s_2 controls the strength of the curvature and thus the validity of the gravity approximation. The limit

$$s_1 \to 0 \tag{12}$$

is particularly interesting known as the *extremal* limit with the seemingly odd interpretation that the region of spacetime behind the horizon has been removed. This limit can be understood better by taking a further *near horizon* limit, this means focussing only on the region around $r = 0$. I claim that the result of this computation is

$$\begin{aligned} ds_{10}^2 &= L^2\big(\frac{dr^2}{r^2} + r^2 ds_{1,3}^2\big) + L^2 d\Omega_5^2 && (13) \\ F_{(5)} &= \mathcal{F}_{(5)} + *\mathcal{F}_{(5)} && \\ \mathcal{F}_{(5)} &= 16\pi(\alpha')^2 N \mathrm{vol}(S^5). && (14) \end{aligned}$$

Here α' comes from the normalization of the string world sheet action and is related to the *string length* (l_s) by $\alpha' = l_s^2$. This is in turn related to the physical gravitational coupling by[4]

$$g_s l_s^4 = \kappa_{10}\,. \tag{15}$$

One also finds the relation

$$L^4 = 4\pi g_s N \alpha'^2 \tag{16}$$

and

$$\int_{S^5} \mathcal{F}_{(5)} = (4\pi\alpha')^2 N\,. \tag{17}$$

A key property of Dp-branes in string is that their gravitational attraction is equal in magnitude to their repulsion due to their coupling to the RR gauge fields[5] [6]. When this gravitional coupling is large, we cannot ignore the backreaction of these Dp-branes on spacetime and we must study the gravity solution corresponding to these sources.

In these lectures we will only mention supersymmetry quite sparingly. Supersymmetry allows one to perform precision tests of holography but in fact is not essential to the general priniciples. So here we note without proof that this D3-brane solution preserves 16 of the 32 supercharges of IIB supergravity. This is known as a *half-BPS* solution. Without going into too many details, we mention that an irreducible representation of the Clifford algebra in ten dimensions has 16 components, this is the minimal dimension of a spinor in ten dimensions. Type II supergravity has thus has $2 \times 16 = 32$ supercharges. As is familiar from our QFT courses, the irreducible representation of the Clifford algebra in four dimensions has four real components then some basic undergraduate math tells us that $4 \times 4 = 16$. We of course then immediately suspect that the field theory living on the world-volume of the D3-brane must possess $\mathcal{N} = 4$ supersymmetry.

If we are to truly limit ourselves to a study of gravity (coupled to various matter) we should only include solutions of the field equations which are suitable regular. Of course certain types of singularities are allowed but this is a subtle topic. Most familiarly, singularities which are cloaked by a horizon, are considered to be *physical.* Even allowing for such singularities, one

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[4] I like, and have checked, the normalization discussion in section 1 of the great review article [20]

[5] This is due to the property known as BPS. I will not explain technically the origin of this crucial property and will thus try to limit referencing it

must provide a strategy for alleviating them and that strategy in general involves invoking the more general spectrum of string theory. Naively, the harmonic function $h(r)$ in this solution appears to blow up at $r = 0$ but further insepction reveals that in fact the solution is perfectly regular there. This property is shared by Dp branes for only special values of p.

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